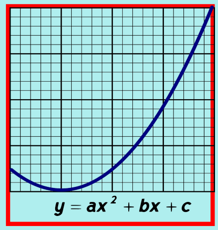
Math 25

Fall 2017

Lecture 6



The table below gives the cost to the vendor and the selling price for each item.

	HB	HD	Solas
Cost	\$2	\$4	\$1
Selling Price	\$4	\$5	\$2

The vendor sold a total of 150 items, spent \$250, made \$440 in revenue. How many of each?

$x \to$ HB

$y \to$ HD

$z \to$ Solas

$$\begin{cases} x + y + z = 150 \\ 2x + 4y + 1z = 250 \\ 4x + 5y + 2z = 440 \end{cases}$$

① Matrices

$$\underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 1 \\ 4 & 5 & 2 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 150 \\ 250 \\ 440 \end{bmatrix}}_{b}$$

$$x = A^{-1}b$$

Let's look for A^{-1}

$$\left[\begin{array}{ccc:ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 2 & 4 & 1 & 0 & 1 & 0 \\ 4 & 5 & 2 & 0 & 0 & 1 \end{array}\right]$$

$(-2)R1 + R2 \to R2$

$(-4)R1 + R3 \to R3$

$$\left[\begin{array}{ccc:ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 2 & -1 & -2 & 1 & 0 \\ 0 & 1 & -2 & -4 & 0 & 1 \end{array}\right] \to$$

$R2 \leftrightarrow R3$

$$\left[\begin{array}{ccc:ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & -4 & 0 & 1 \\ 0 & 2 & -1 & -2 & 1 & 0 \end{array}\right]$$

$(-2)R2 + R3 \to R3$, $(-1)R2 + R1 \to R1$

$$\left[\begin{array}{ccc:ccc} 1 & 0 & 3 & 5 & 0 & -1 \\ 0 & 1 & -2 & -4 & 0 & 1 \\ 0 & 0 & 3 & 6 & 1 & -2 \end{array}\right]$$

$(-1)R3 + R1 \to R1$

$$\left[\begin{array}{ccc:ccc} 1 & 0 & 0 & -1 & -1 & 1 \\ 0 & 1 & -2 & -4 & 0 & 1 \\ 0 & 0 & 3 & 6 & 1 & -2 \end{array}\right]$$

$(3)R2 \to R2$ $(2)R3 \to R3$

$$\left[\begin{array}{ccc:ccc} 1 & 0 & 0 & -1 & -1 & 1 \\ 0 & 3 & -6 & -12 & 0 & 3 \\ 0 & 0 & 6 & 12 & 2 & -4 \end{array}\right]$$

$R3 + R2 \to R2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & -1 & 1 \\ 0 & 3 & -6 & -12 & 0 & 3 \\ 0 & 0 & 6 & 12 & 2 & -4 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & -1 & 1 \\ 0 & 3 & 0 & 0 & 2 & -1 \\ 0 & 0 & 6 & 12 & 2 & -4 \end{array}\right]$$

$R2 \div 3 \to R2$, $R3 \div 6 \to R3$

$$\underset{I \qquad\qquad A^{-1}}{\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & -1 & 1 \\ 0 & 1 & 0 & 0 & \frac{2}{3} & \frac{-1}{3} \\ 0 & 0 & 1 & 2 & \frac{1}{3} & \frac{-2}{3} \end{array}\right]} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 & -1 & 1 \\ 0 & \frac{2}{3} & \frac{-1}{3} \\ 2 & 1/3 & -2/3 \end{bmatrix} \begin{bmatrix} 150 \\ 250 \\ 440 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \underset{3\times3}{\begin{bmatrix} -1 & -1 & 1 \\ 0 & \frac{2}{3} & \frac{-1}{3} \\ 2 & 1/3 & -2/3 \end{bmatrix}} \underset{3\times1}{\begin{bmatrix} 150 \\ 250 \\ 440 \end{bmatrix}} = \underset{3\times1}{\begin{bmatrix} -1\cdot150 - 1\cdot250 + 1\cdot440 \\ 0\cdot150 + \frac{2}{3}\cdot250 - \frac{1}{3}440 \\ 2\cdot150 + \frac{1}{3}\cdot250 - \frac{2}{3}440 \end{bmatrix}} = \begin{bmatrix} 40 \\ 20 \\ 90 \end{bmatrix}$$

$\frac{500}{3} - \frac{440}{3} = \frac{60}{3} = 20$

$300 + \frac{250}{3} - \frac{880}{3}$

$= 300 - \frac{630}{3} = 300 - 210 = 90$

40 HB,
20 HD,
90 Sodas

Find the determinant of the coef. matrix for the system given below:

$$\begin{cases} x + 2y - 3z = 10 \\ 3x - y + 4z = -5 \\ 4x + y + z = 5 \end{cases} \qquad \begin{bmatrix} 1 & 2 & -3 \\ 3 & -1 & 4 \\ 4 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ -5 \\ 5 \end{bmatrix}$$

$$\begin{vmatrix} 1 & 2 & -3 \\ 3 & -1 & 4 \\ 4 & 1 & 1 \end{vmatrix} = 1\begin{vmatrix} -1 & 4 \\ 1 & 1 \end{vmatrix} - 2\begin{vmatrix} 3 & 4 \\ 4 & 1 \end{vmatrix} + (-3)\begin{vmatrix} 3 & -1 \\ 4 & 1 \end{vmatrix}$$

Expand by first row

$$= 1(-1-4) - 2(3-16) - 3(3 - -4)$$

$$= -5 + 26 - 21 = \boxed{0}$$

Since A, the coef. matrix, has the det. value of 0, $|A| = 0$, then A^{-1} does not exist.

So to solve the system, we cannot use matrix method.

Also we can use Cramer's rule.

The system may have no solution or infinitely many solutions.